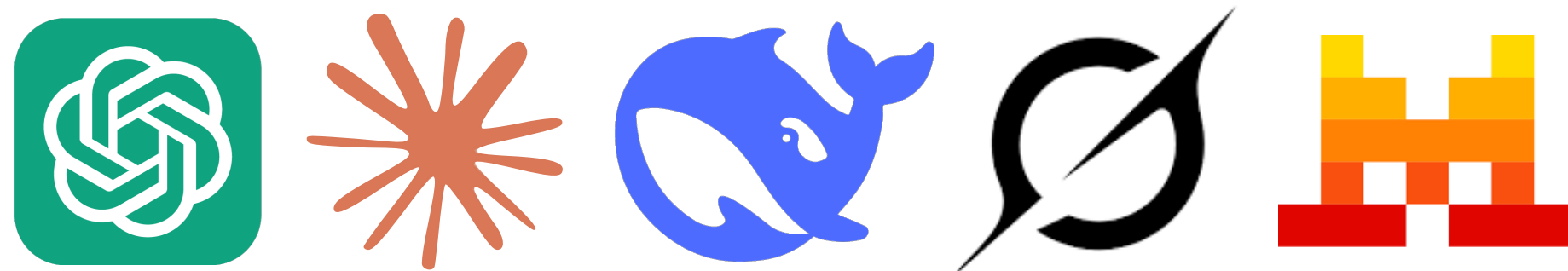
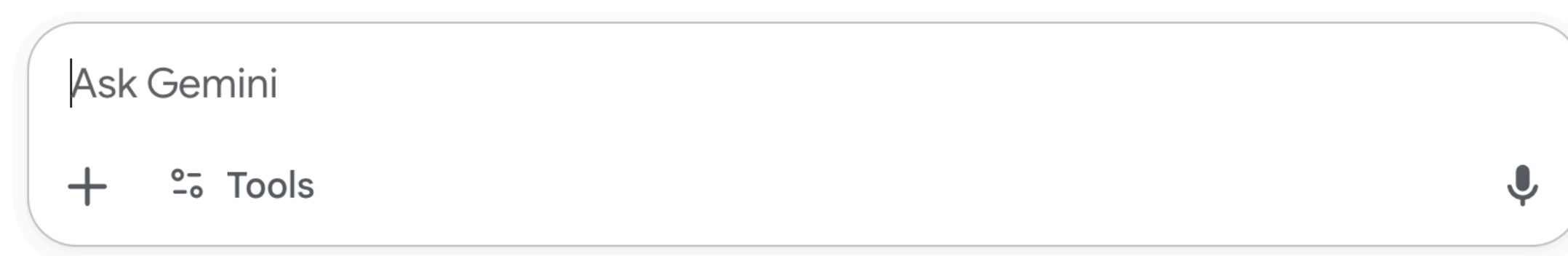


Ph 220: Lecture 12

Generative Quantum Advantage

Generative Models

- **Definition (informal):** **Generative model** — A model that can learn to generate new outputs from some probability distribution.



1010011000111001
1000101000011101
11100110101
010100100110
1110001001001001
100101001
10010010001001

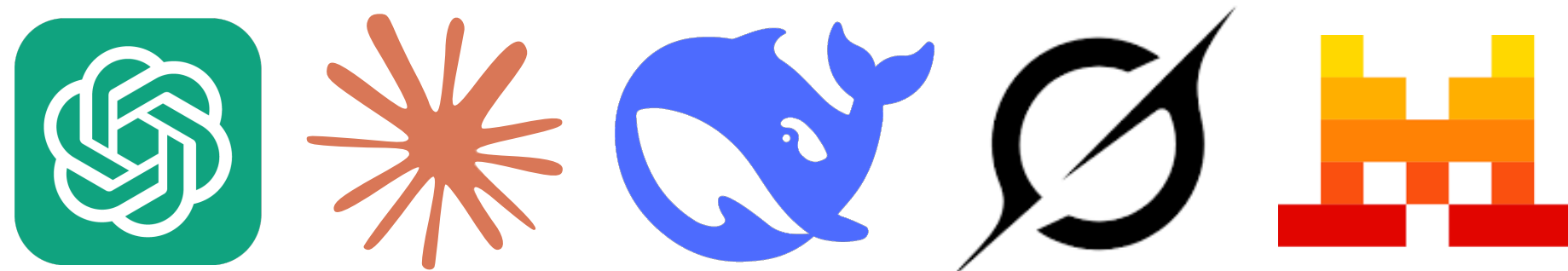
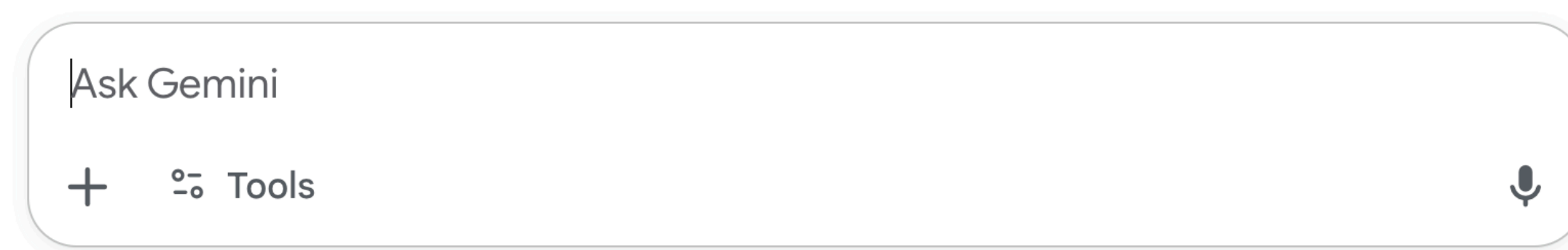
**Training
Dataset**

Input:
0010010100

Output:
1101001110

Generative Task

- **Definition:** Given a dataset of (x_i, y_i) sampled from unknown $p(y | x)$, learn to generate **new** y for **any given** x .



1010011000111001
1000101000011101
11100110101
010100100110
1110001001001001
100101001
10010010001001

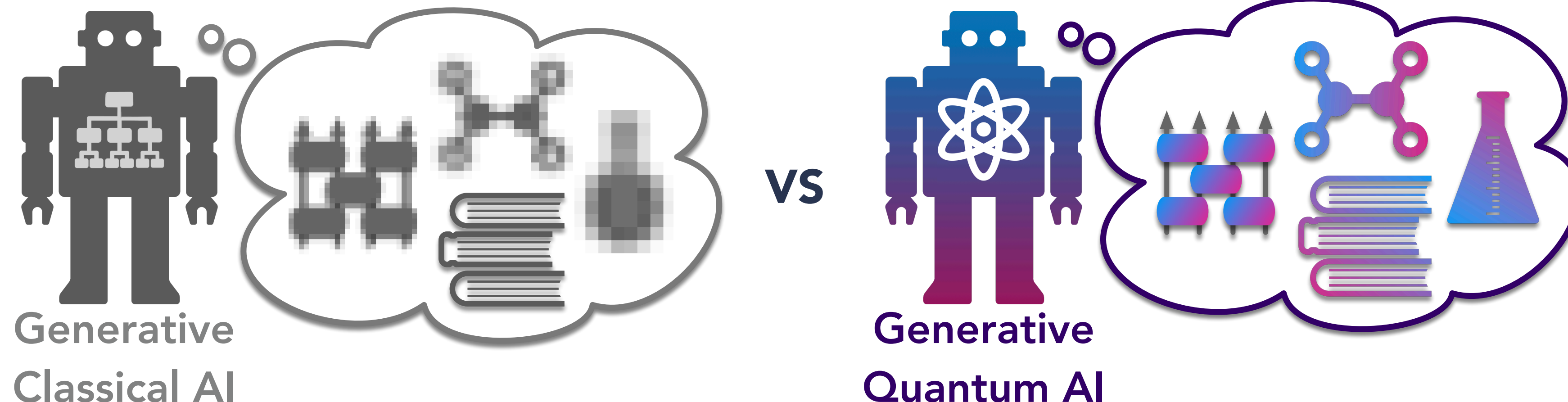
**Training
Dataset**

Input:
0010010100

Output:
1101001110

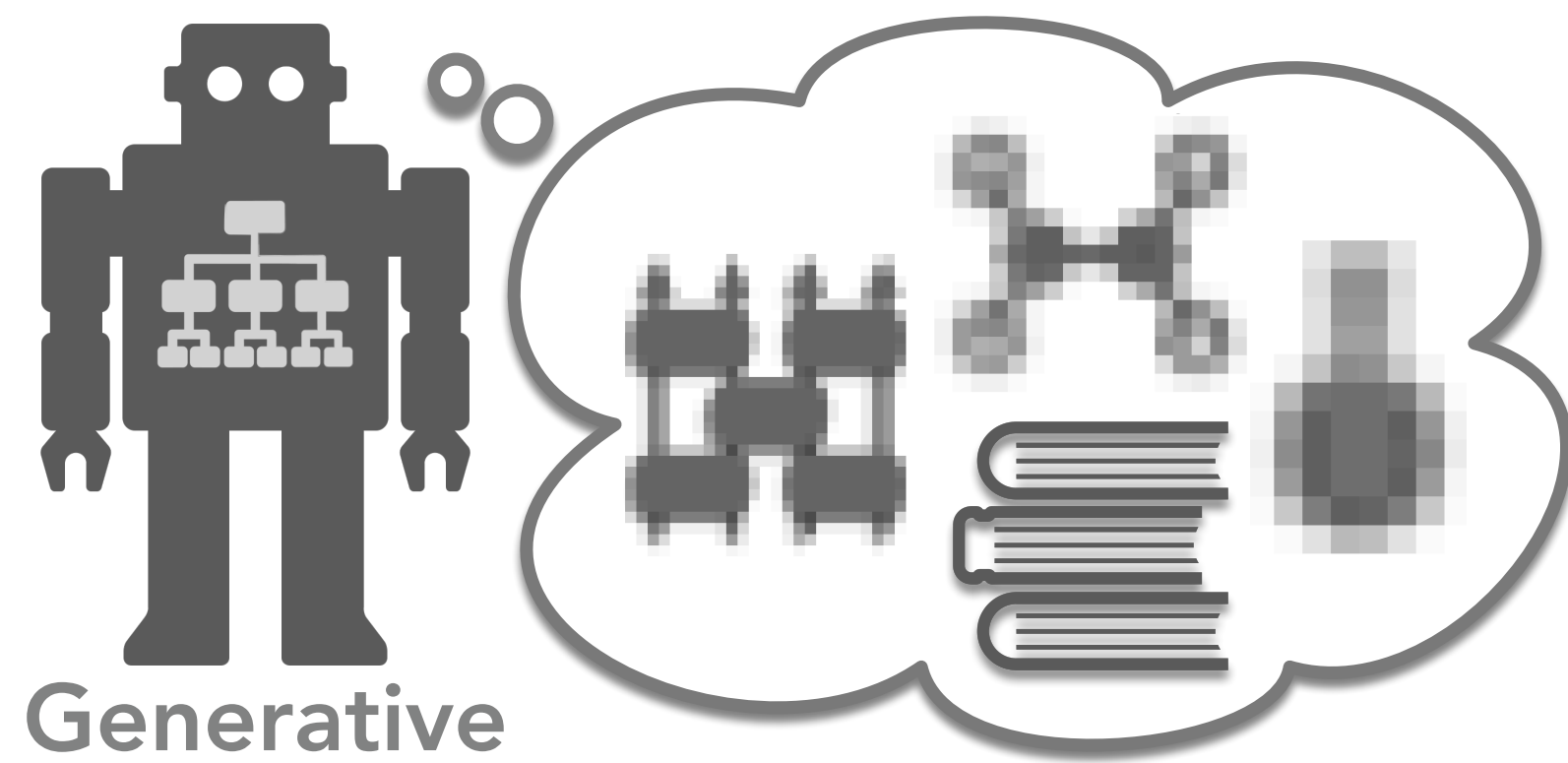
Generative Quantum Advantage

- Definition (informal): **Generative quantum advantage** — A quantum computer can learn to generate the desired outputs



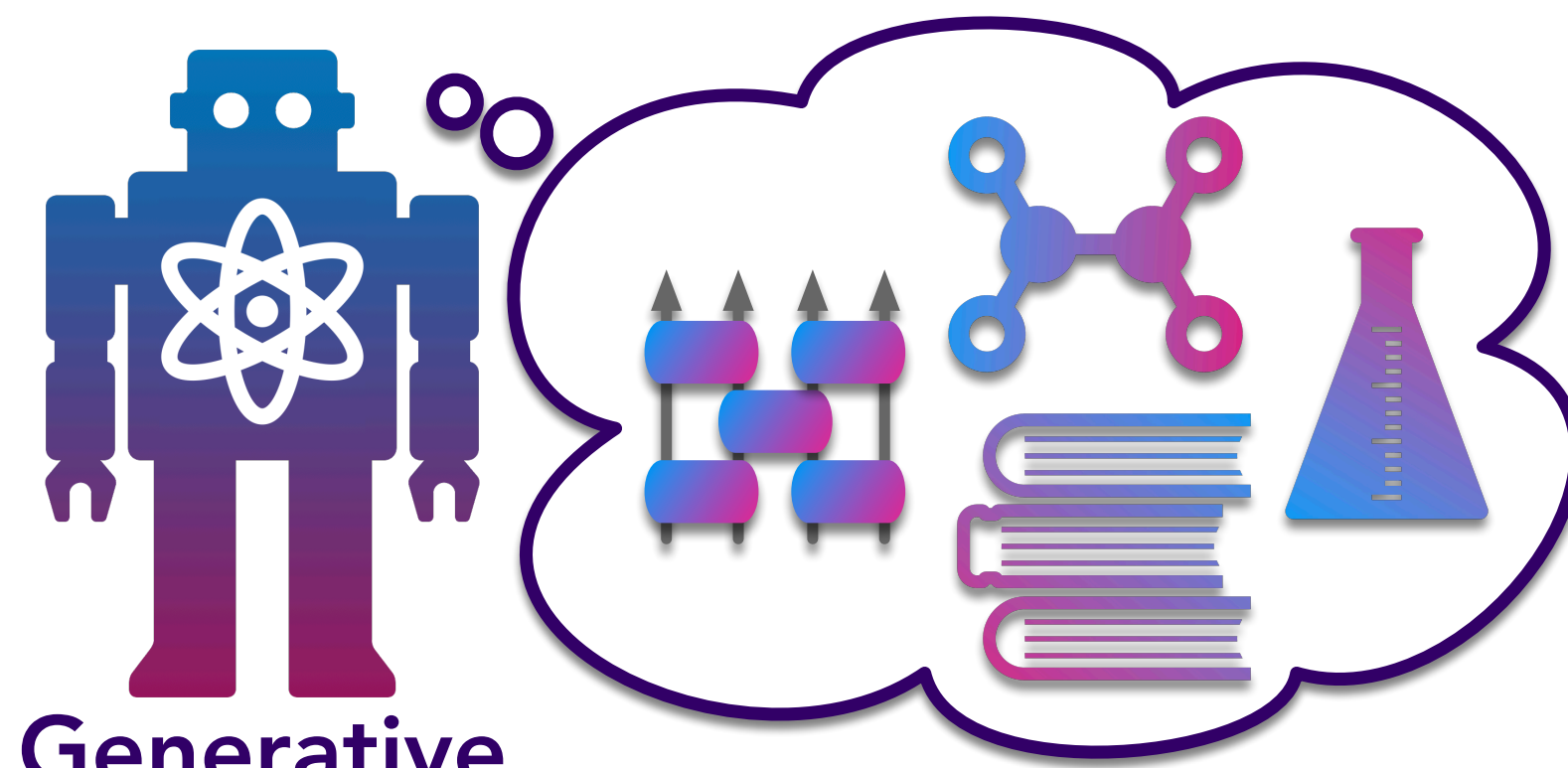
Generative Quantum Advantage

- Definition (informal): **Generative quantum advantage** — A quantum computer can learn to generate the desired outputs with **reduced** sample complexity



Generative
Classical AI

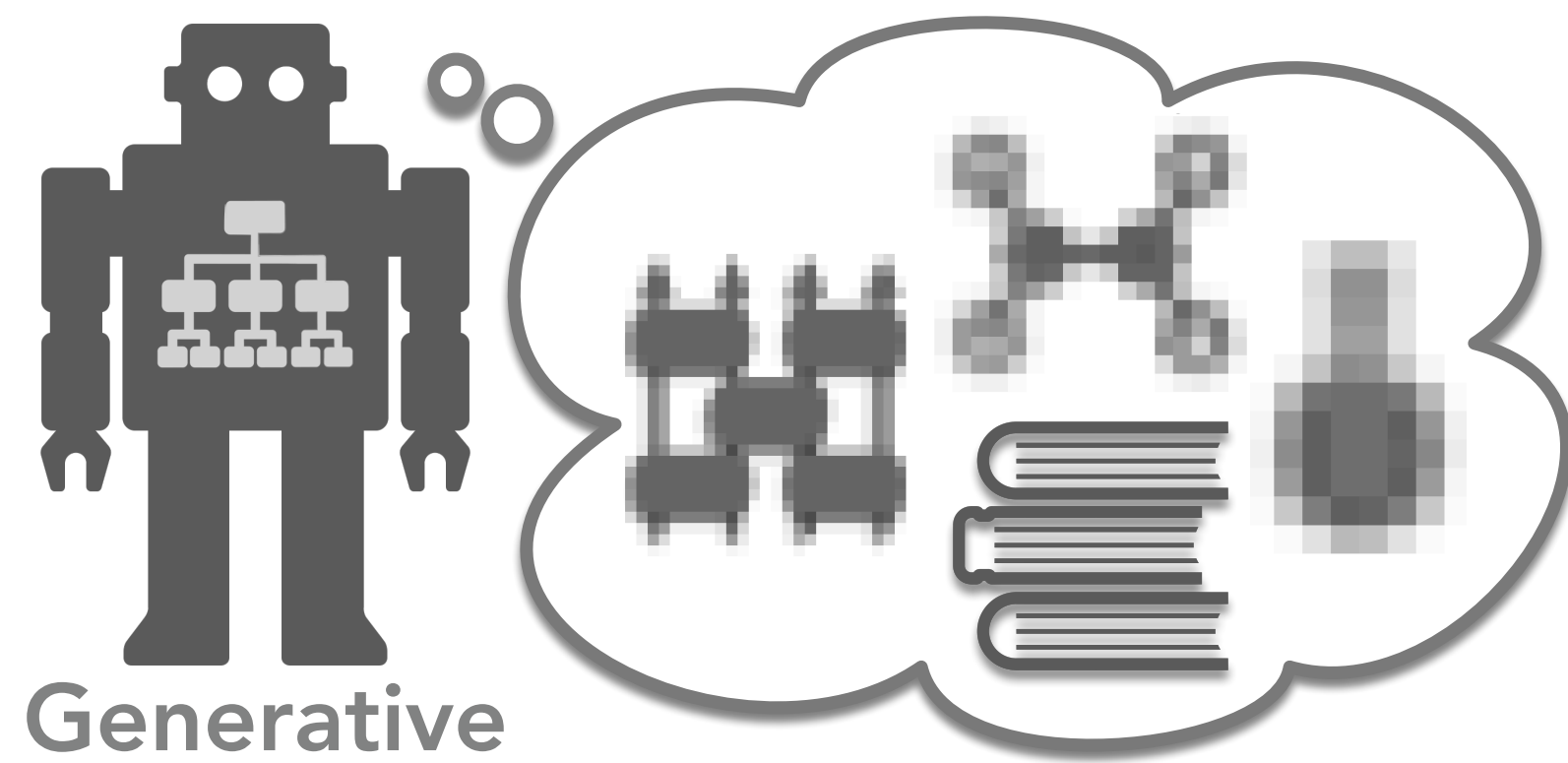
vs



Generative
Quantum AI

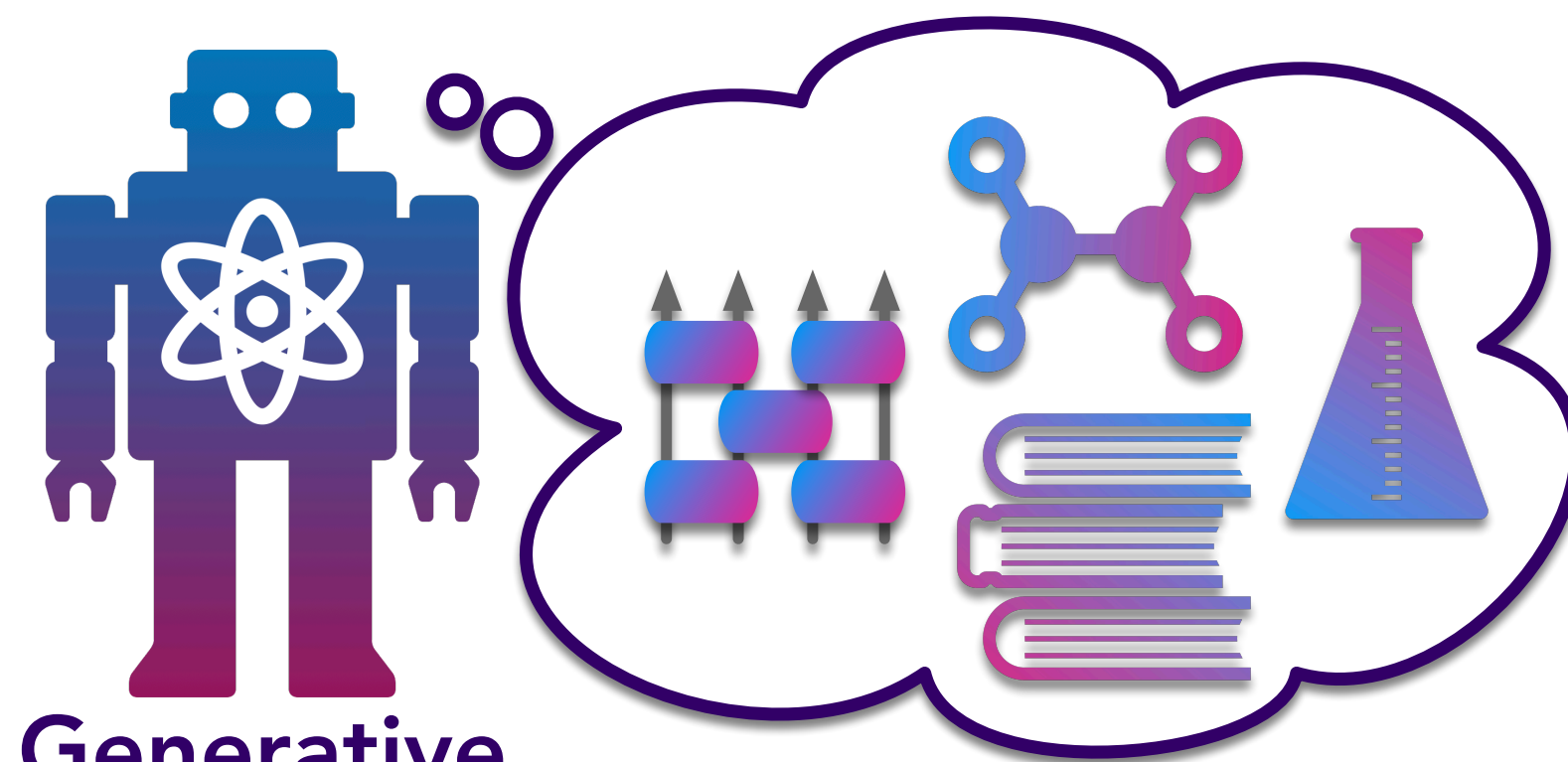
Generative Quantum Advantage

- Definition (informal): **Generative quantum advantage** — A quantum computer can learn to generate the desired outputs with **reduced** sample complexity, **higher** accuracy,



Generative
Classical AI

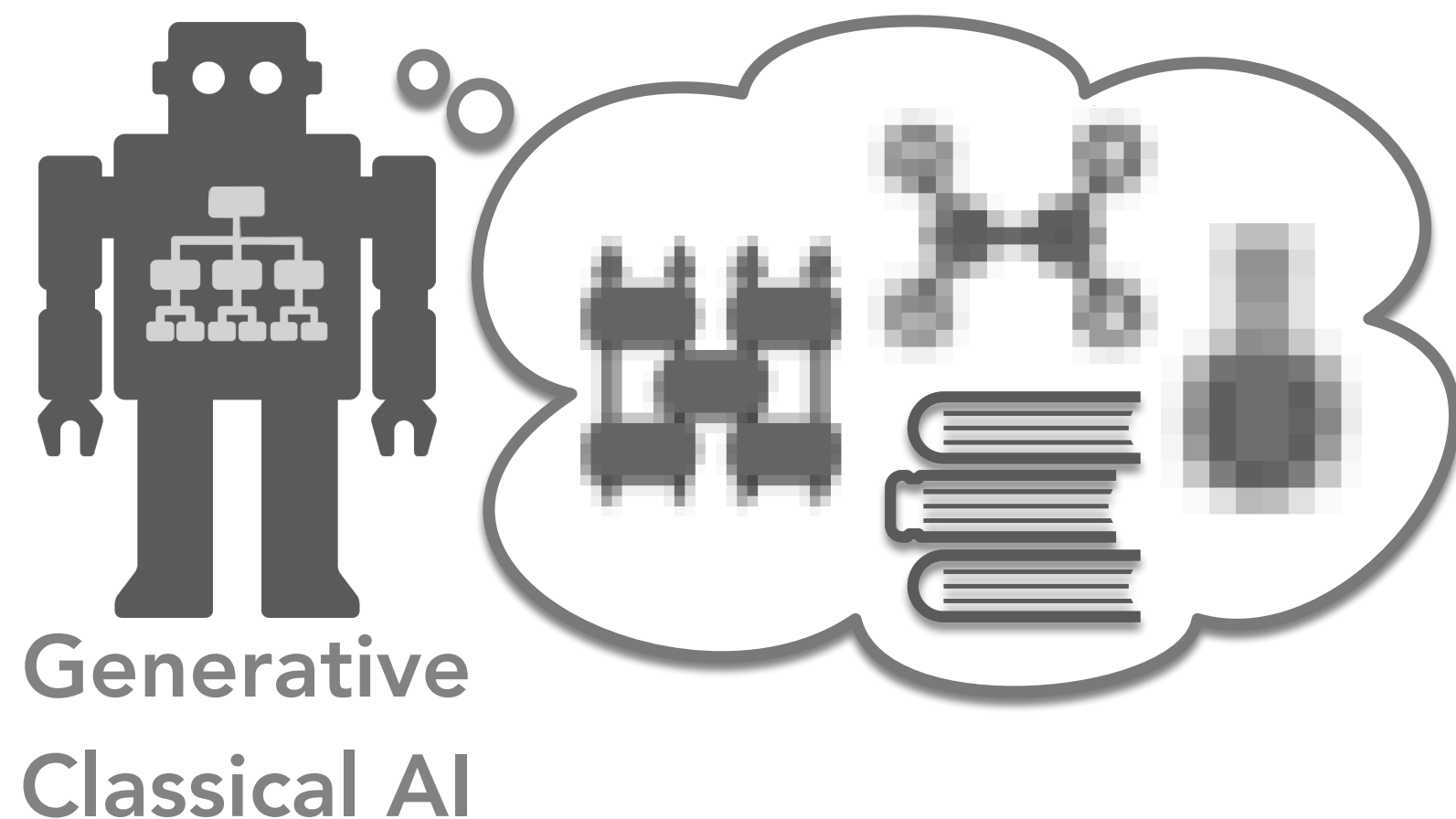
vs



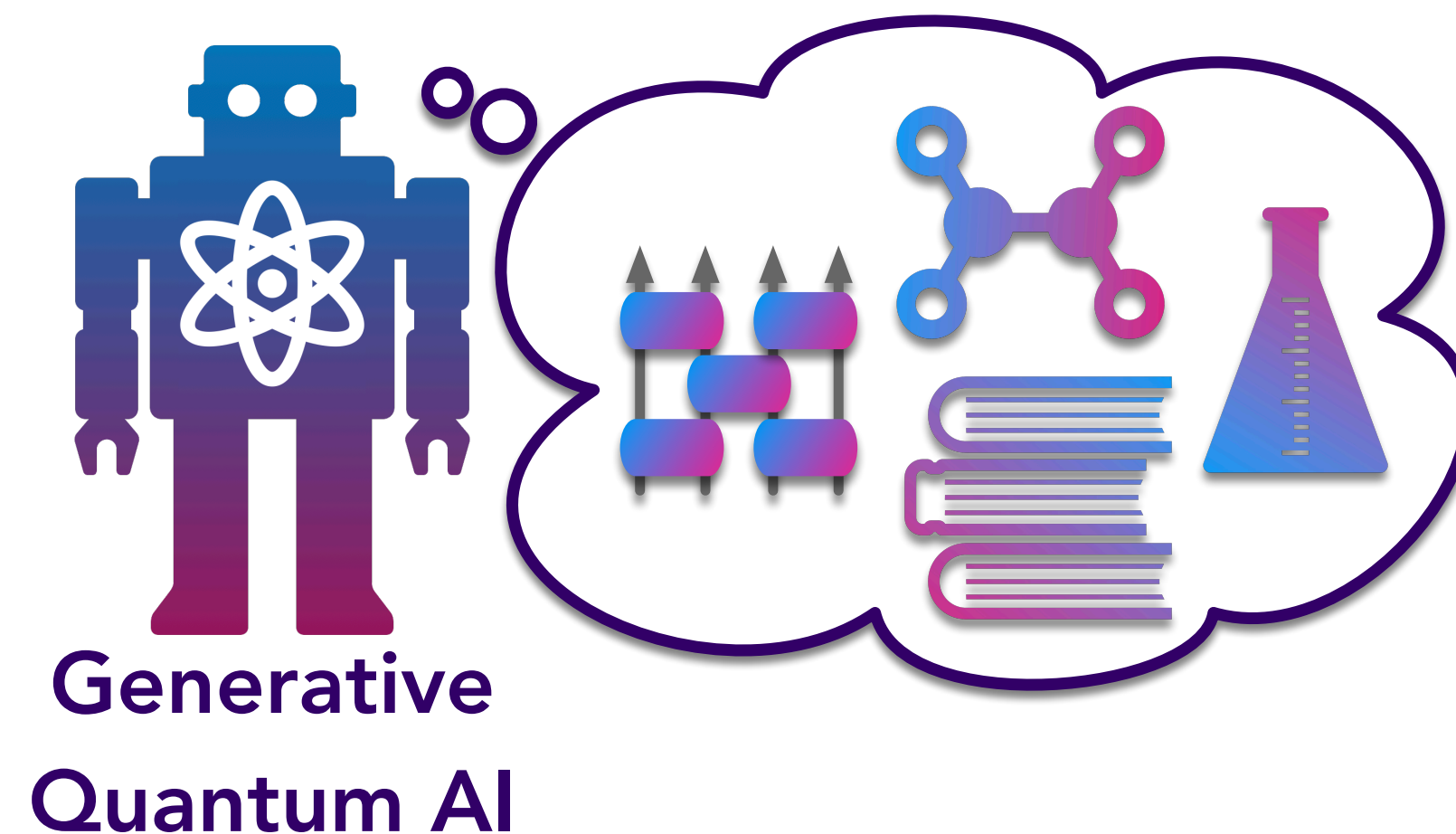
Generative
Quantum AI

Generative Quantum Advantage

- Definition (informal): **Generative quantum advantage** — A quantum computer can learn to generate the desired outputs with **reduced** sample complexity, **higher** accuracy, **faster** learning and/or generation time,

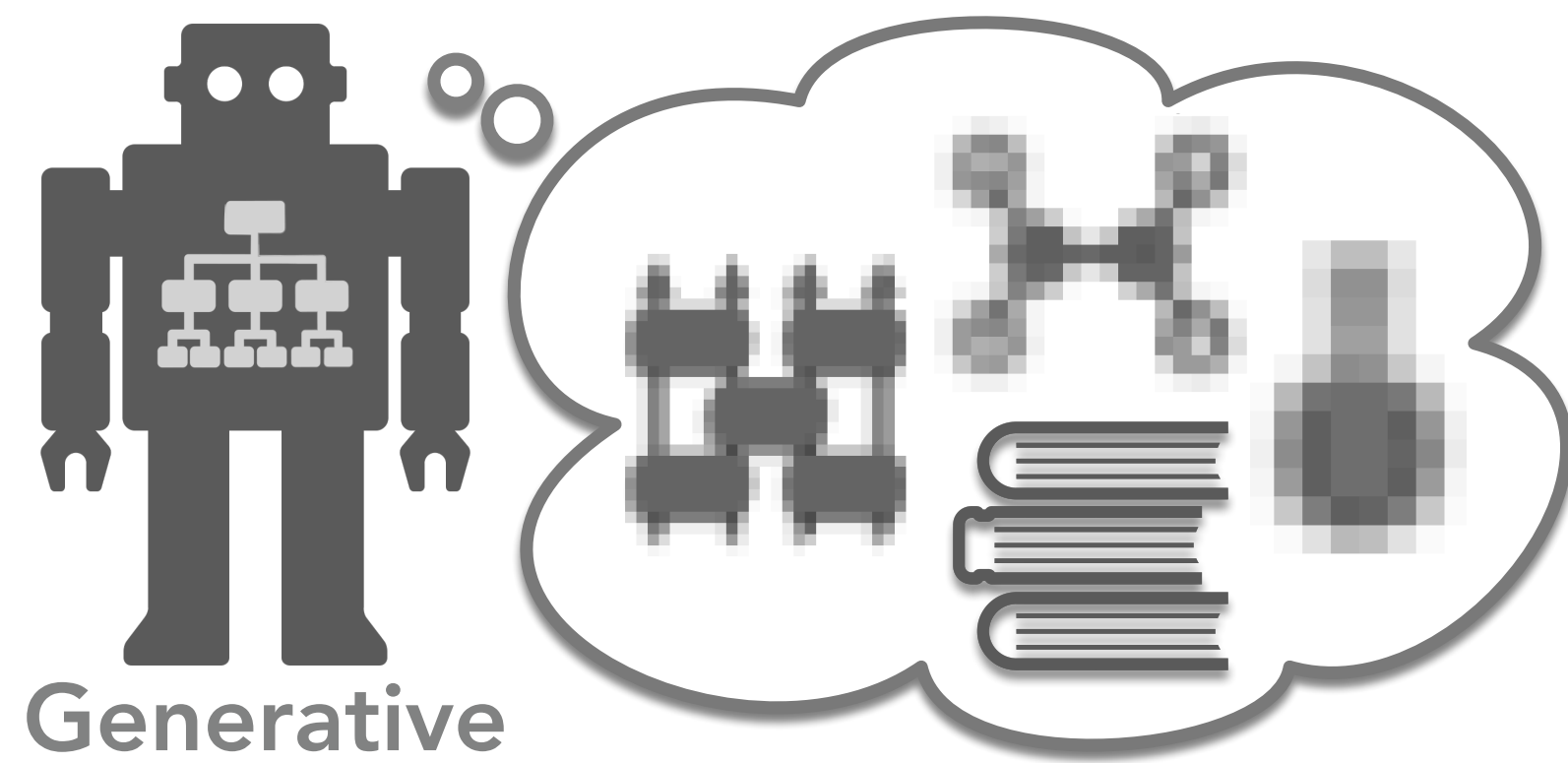


vs



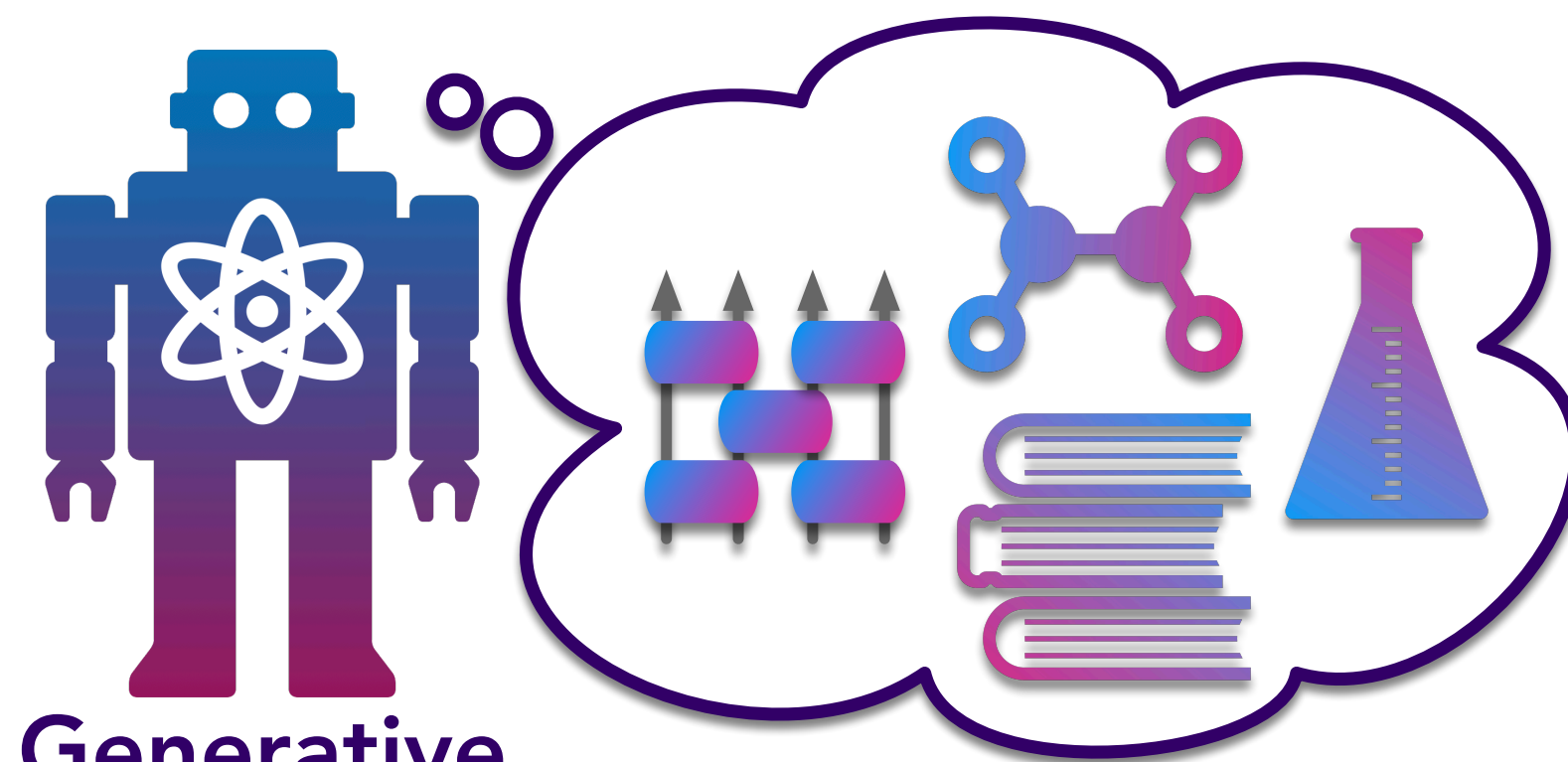
Generative Quantum Advantage

- Definition (informal): **Generative quantum advantage** — A quantum computer can learn to generate the desired outputs with **reduced** sample complexity, **higher** accuracy, **faster** learning and/or generation time, or outputs infeasible for classical computers.

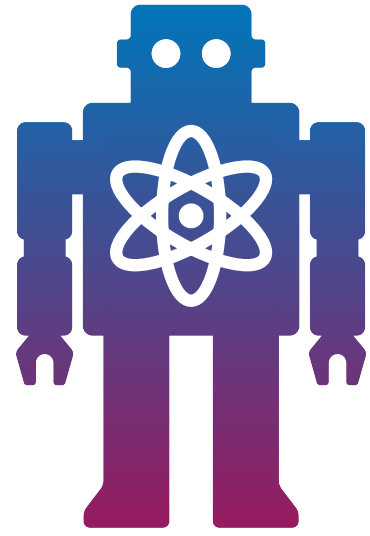


Generative
Classical AI

vs



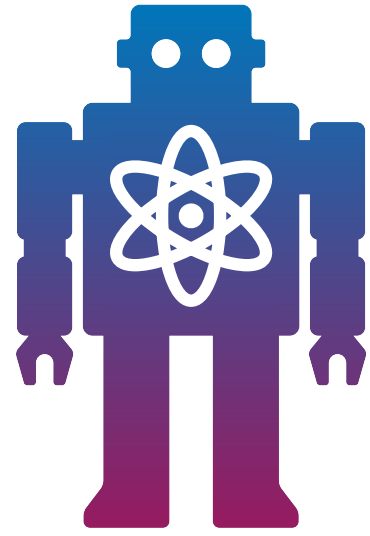
Generative
Quantum AI



Main Question

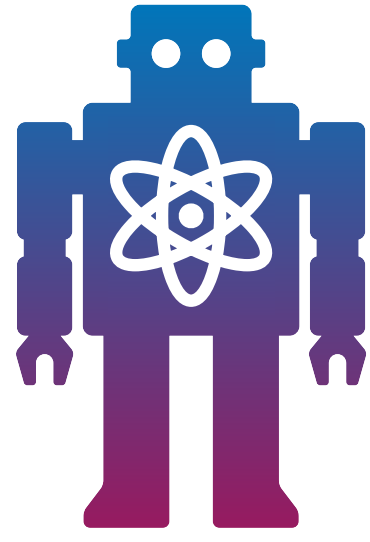
Are there families of unknown distributions $p(y | x)$ mapping classical inputs to classical outputs such that:

- Quantum computers can **efficiently learn** from few samples;
- Quantum computers can **efficiently generate** new outputs;
- Classical computers **cannot efficiently generate** new outputs?



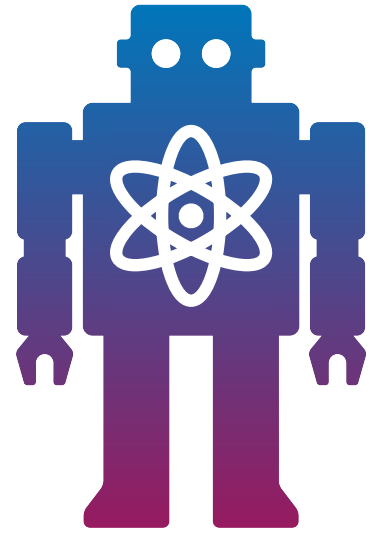
Generative QNNs

- **Task:** Given a dataset of (x_i, y_i) sampled from unknown $p(y | x)$, learn to generate **new** y for **any given** x .



Generative QNNs

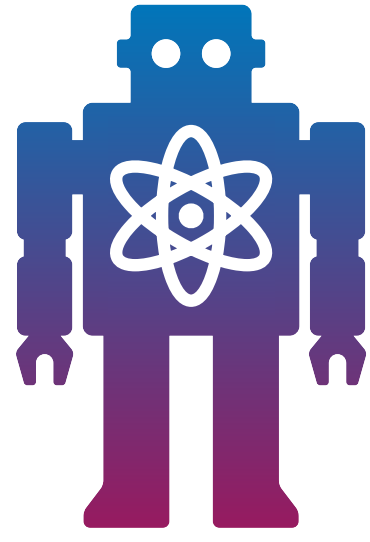
- **Task:** Given a dataset of (x_i, y_i) sampled from unknown $p(y | x)$, learn to generate **new** y for **any given** x .
- **Generative Quantum Neural Networks (QNNs):**
 1. Encode x into quantum state $|\psi_x\rangle$ and measurement basis M_x .
 2. Apply trainable quantum circuit C_β to the state $|\psi_x\rangle$.
 3. Measure $C_\beta|\psi_x\rangle$ in the basis M_x to sample y from $p(y | x; \beta)$.



Main Question

Are there families of unknown distributions $p(y | x)$ mapping classical inputs to classical outputs such that:

- Quantum computers can **efficiently learn** from few samples;
- Quantum computers can **efficiently generate** new outputs;
- Classical computers **cannot efficiently generate** new outputs?



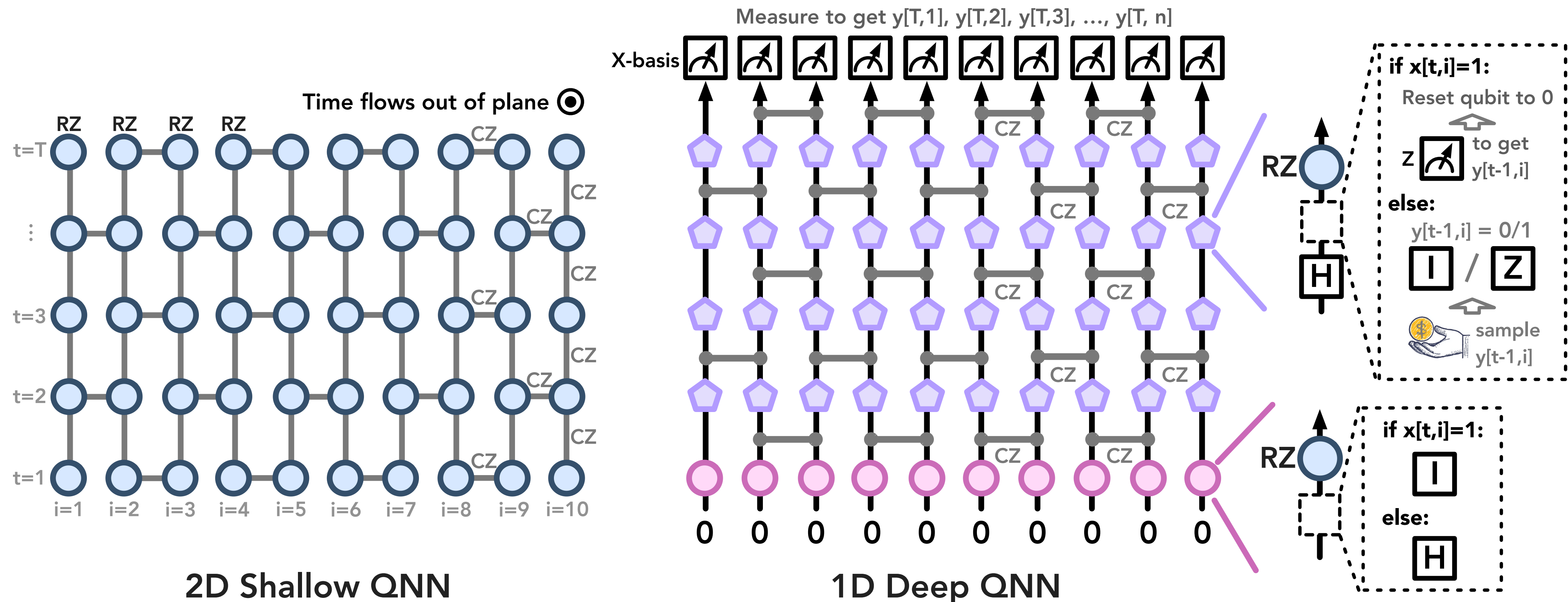
Main Question

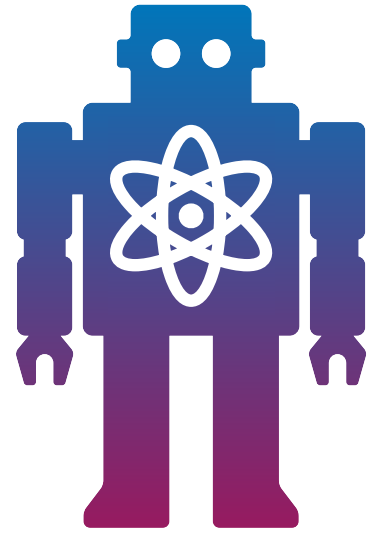
Are there families of unknown distributions $p(y | x)$ mapping classical inputs to classical outputs such that:

- Quantum computers can **efficiently learn** from few samples;
- ✓ • Quantum computers can **efficiently generate** new outputs;
- Classical computers **cannot efficiently generate** new outputs?

Computational Power

- Under standard conjectures, there are **shallow QNNs** that can **generate** distributions **no** poly-time classical algorithm \mathcal{A} can.

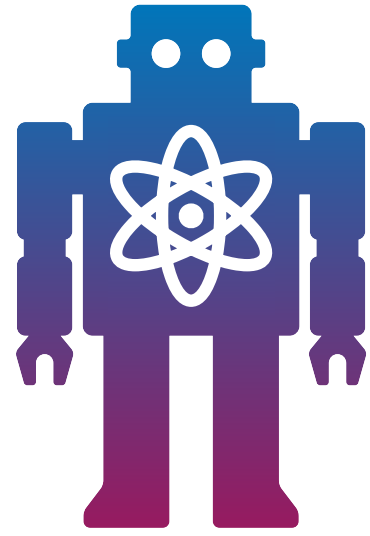




Main Question

Are there families of unknown distributions $p(y | x)$ mapping classical inputs to classical outputs such that:

- Quantum computers can **efficiently learn** from few samples;
- ✓ • Quantum computers can **efficiently generate** new outputs;
- Classical computers **cannot efficiently generate** new outputs?



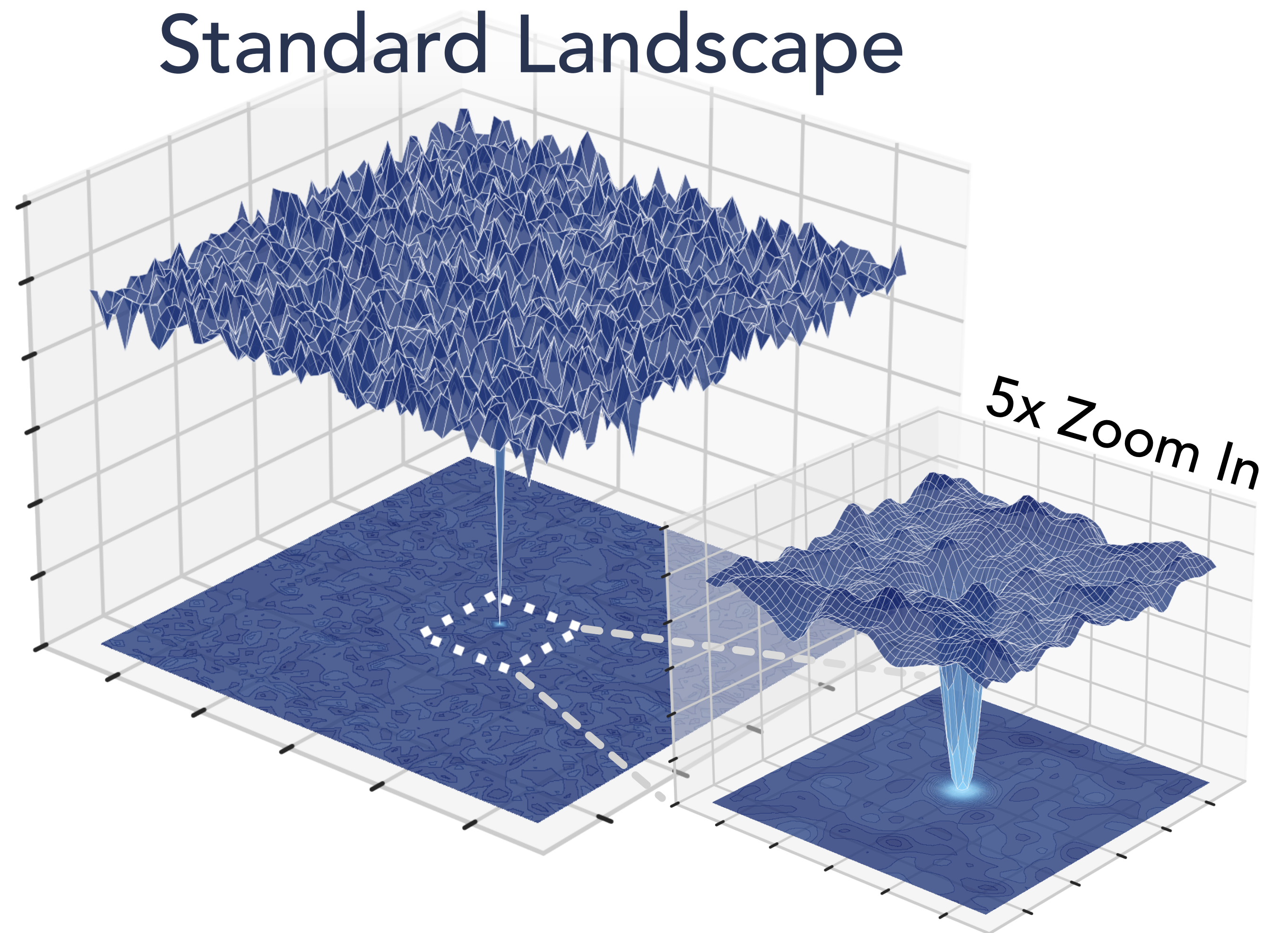
Main Question

Are there families of unknown distributions $p(y | x)$ mapping classical inputs to classical outputs such that:

- Quantum computers can **efficiently learn** from few samples;
- ✓ Quantum computers can **efficiently generate** new outputs;
- ✓ Classical computers **cannot efficiently generate** new outputs?

Bad Loss Landscape

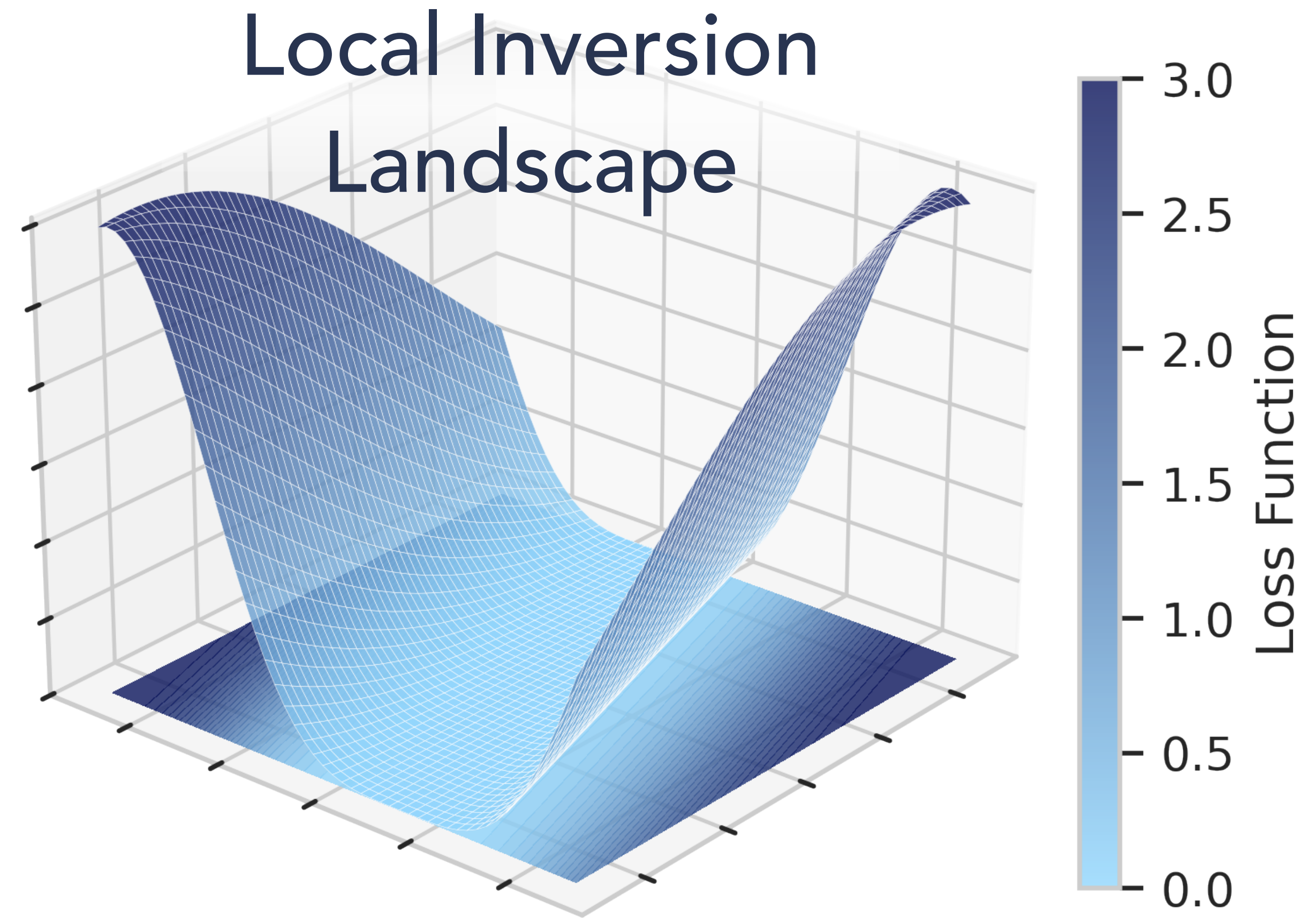
Shallow QNNs
have **extremely
bad** landscape.

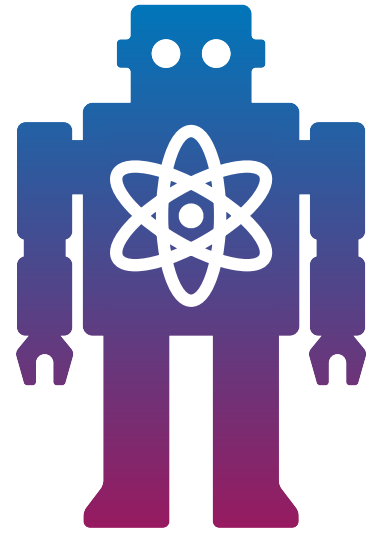


Provably Efficient Learning

Theorem

Any n -qubit shallow QNN
can be learned to ε error
in **$\text{poly}(n, 1/\varepsilon)$ time**.

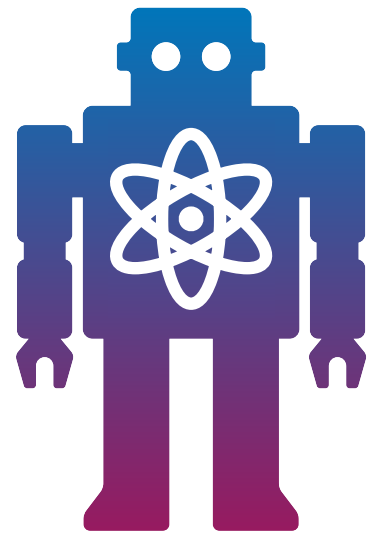




Main Question

Are there families of unknown distributions $p(y | x)$ mapping classical inputs to classical outputs such that:

- Quantum computers can **efficiently learn** from few samples;
- ✓ Quantum computers can **efficiently generate** new outputs;
- ✓ Classical computers **cannot efficiently generate** new outputs?

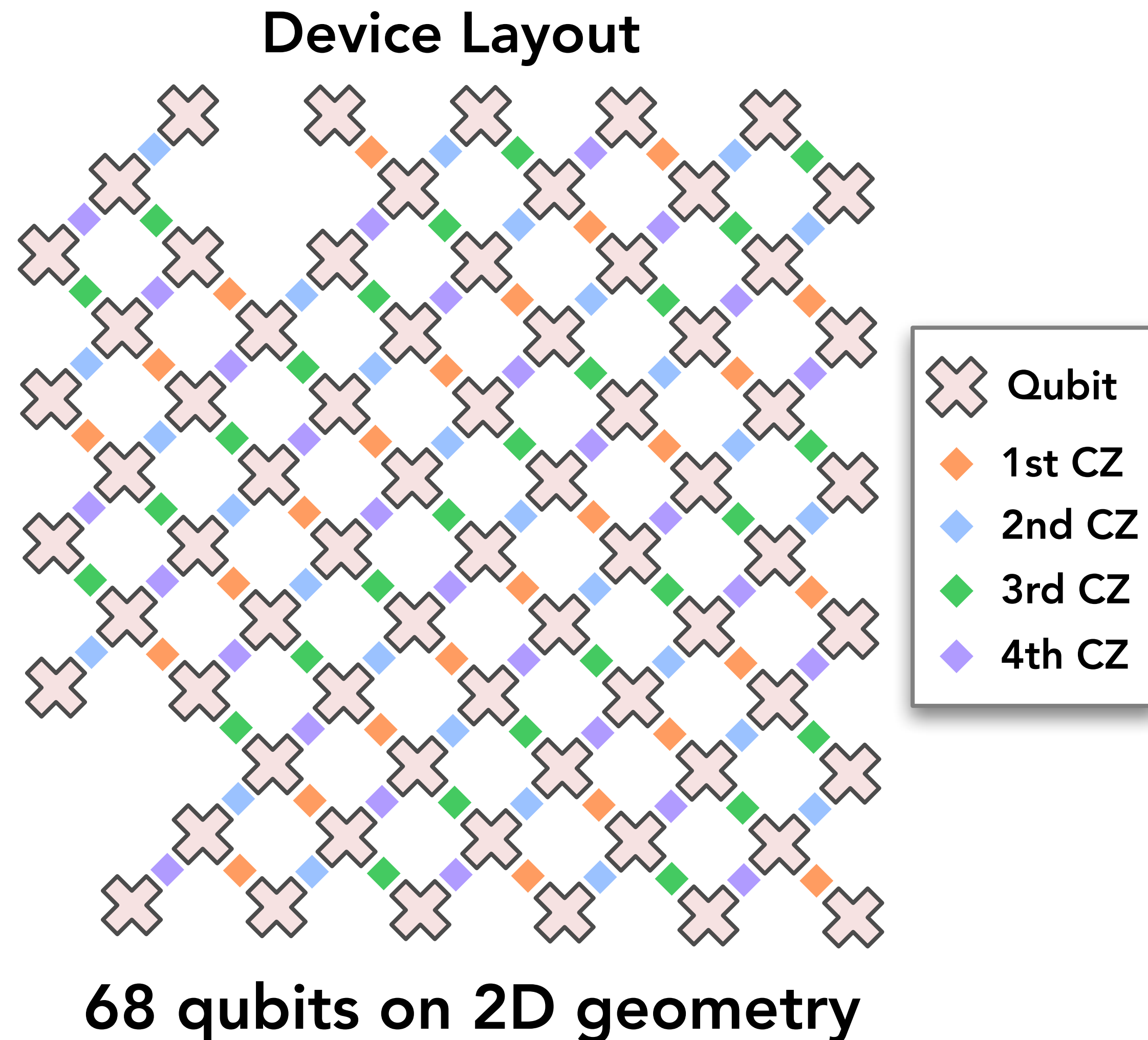


Main Question

Are there families of unknown distributions $p(y | x)$ mapping classical inputs to classical outputs such that:

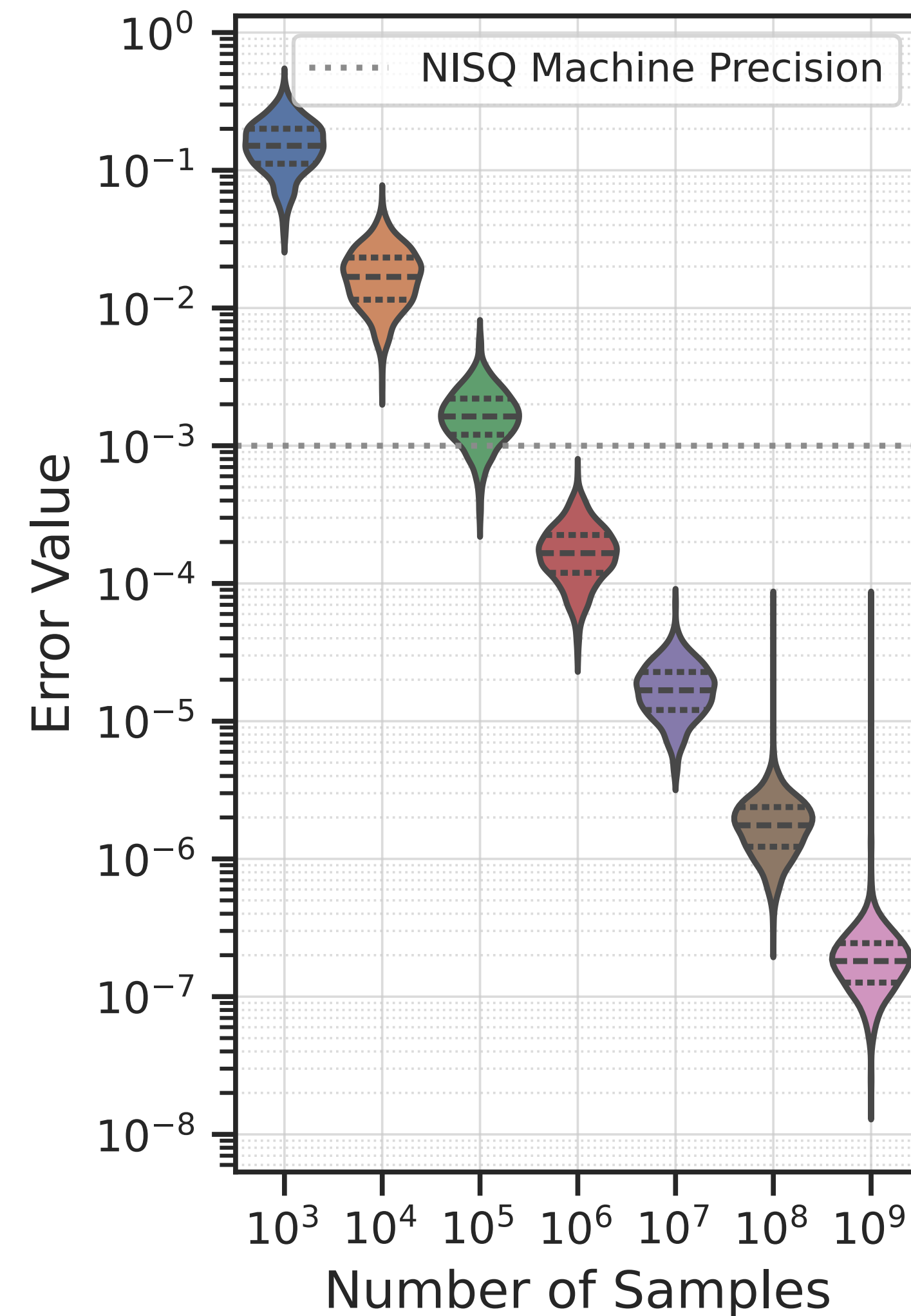
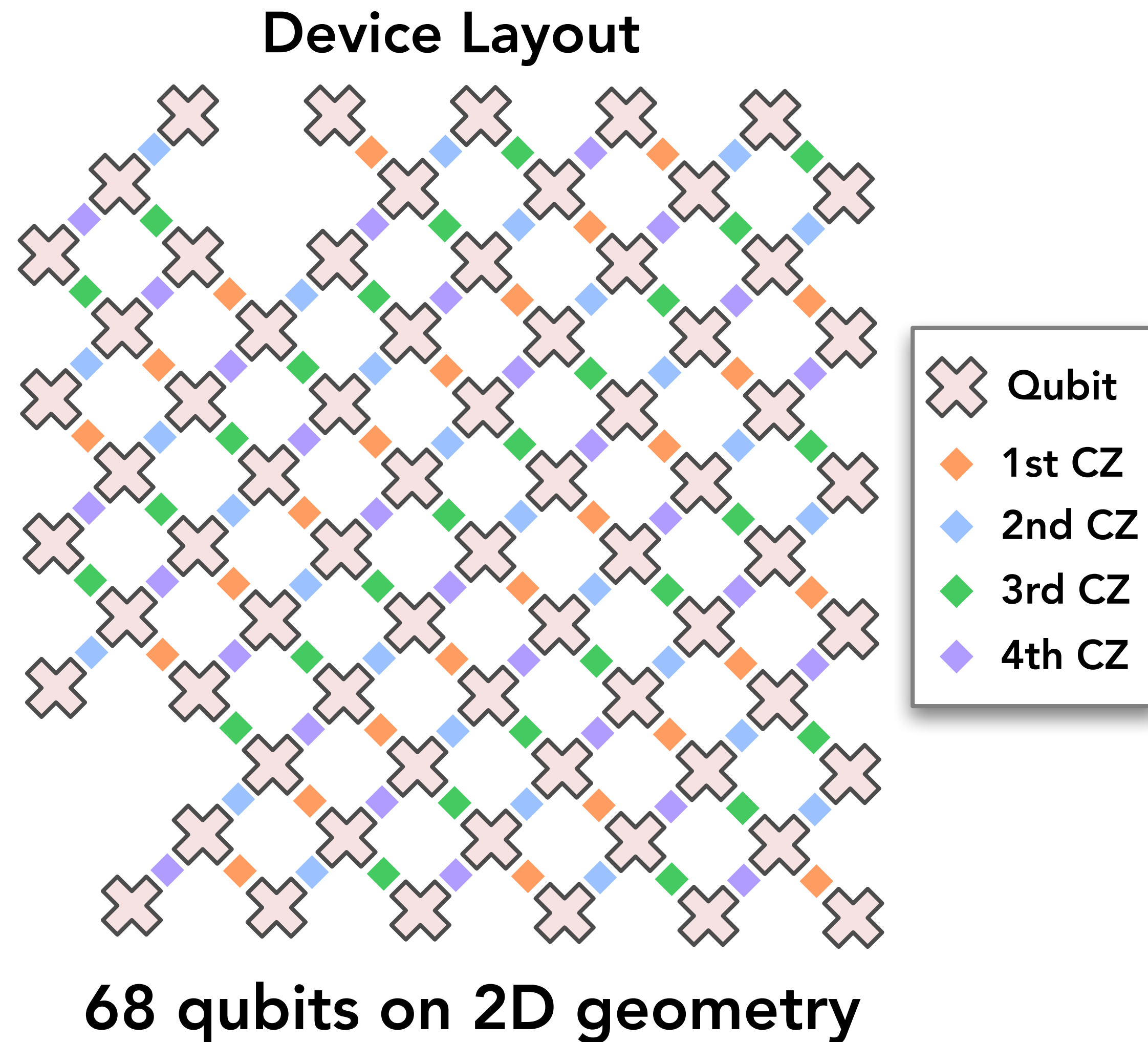
- ✓ Quantum computers can **efficiently learn** from few samples;
- ✓ Quantum computers can **efficiently generate** new outputs;
- ✓ Classical computers **cannot efficiently generate** new outputs?

Experimental Demonstration



- Train **3D** shallow QNN.
- Map **3D** shallow QNN to **2D** deep QNN.
- Generate new output y using **2D** deep QNN.

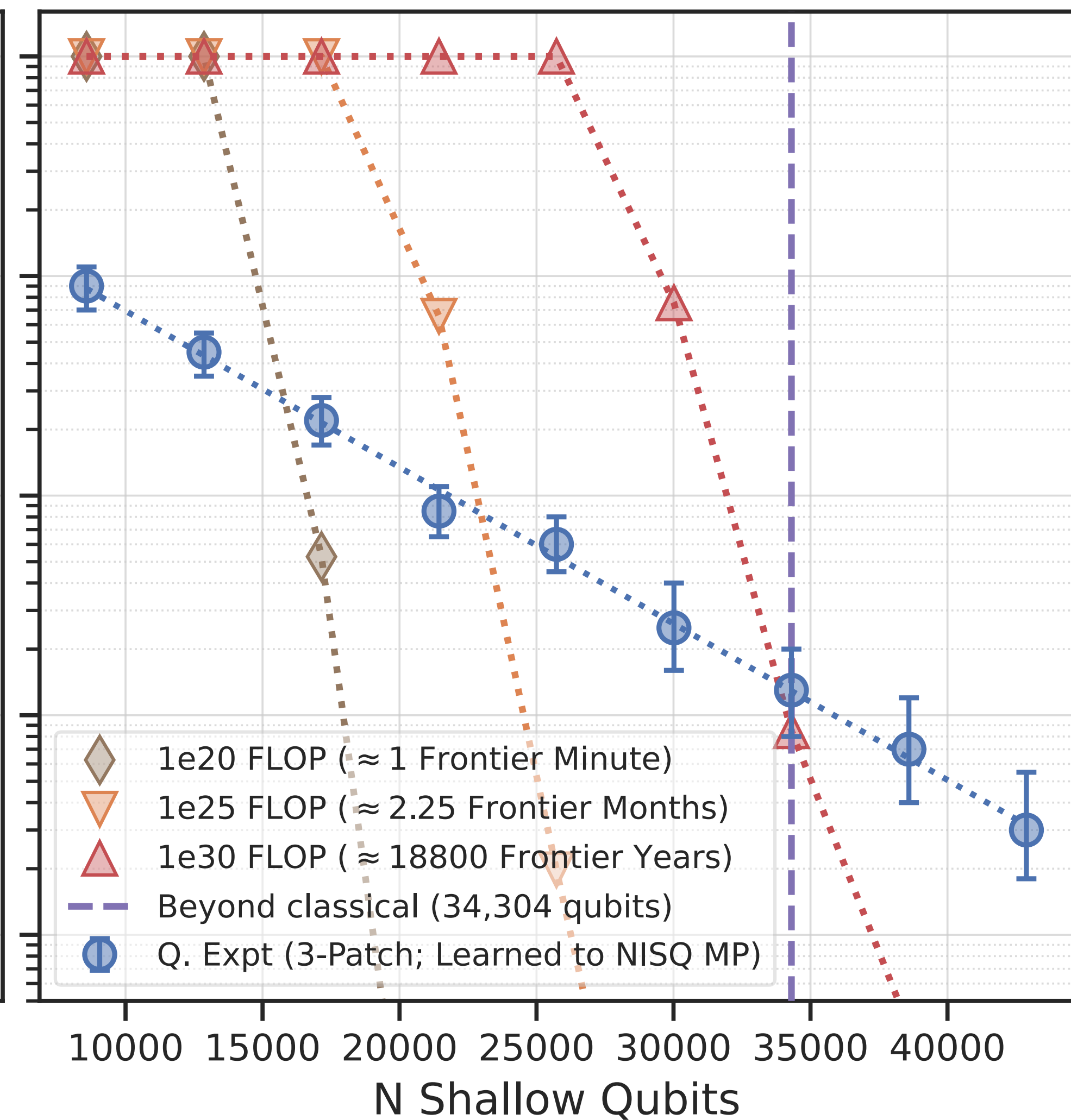
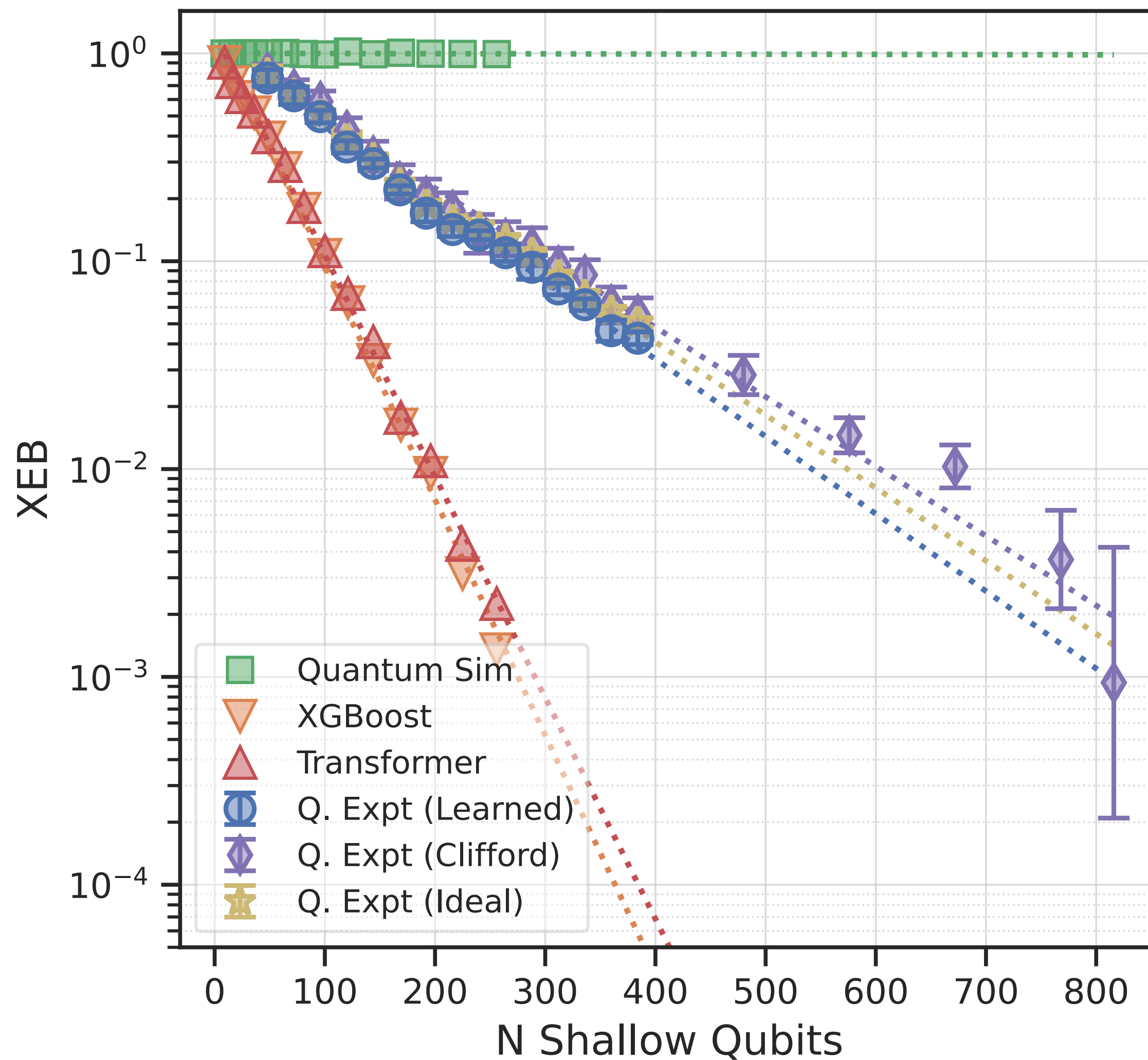
Experimental Demonstration



Experimental Demonstration

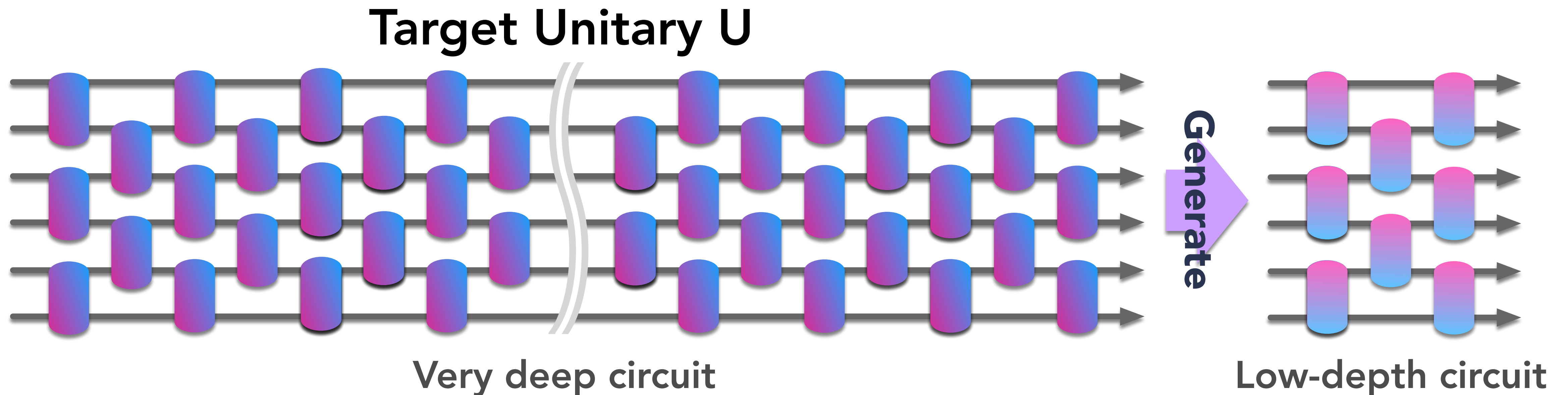
New data collected for smaller shallow QNNs

Data repurposed from Morvan et al., *Nature* (2024)



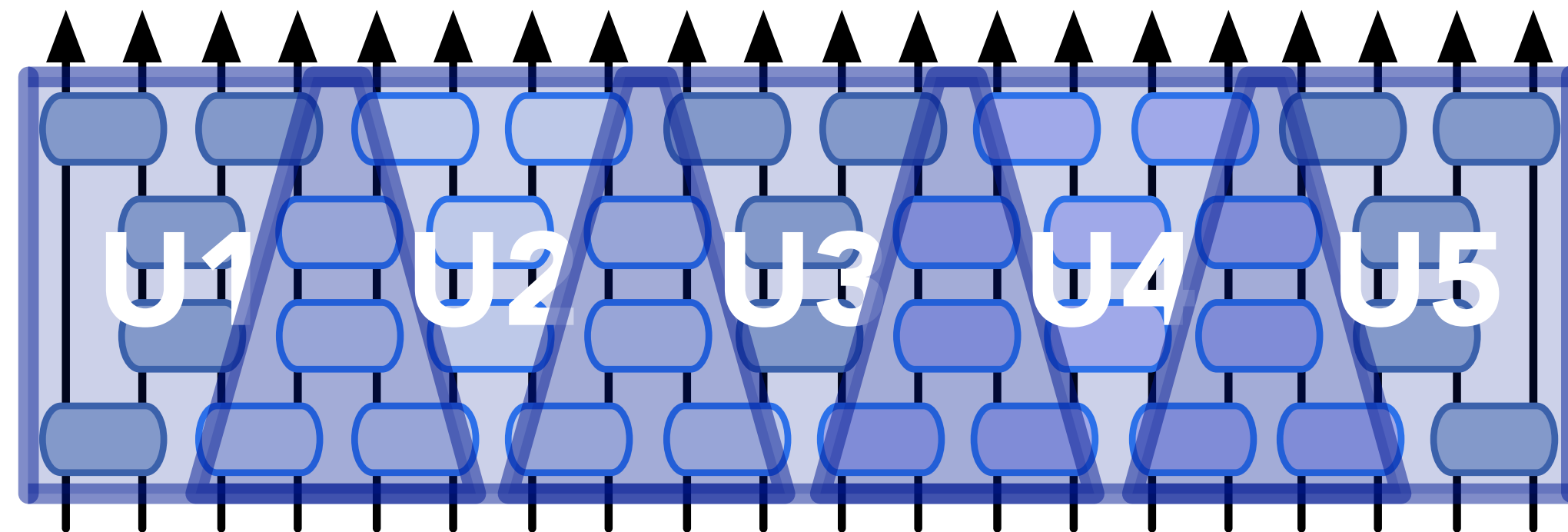
Application: Compressing Dynamics

- We also established generative quantum advantage for the task of compressing physical dynamics.

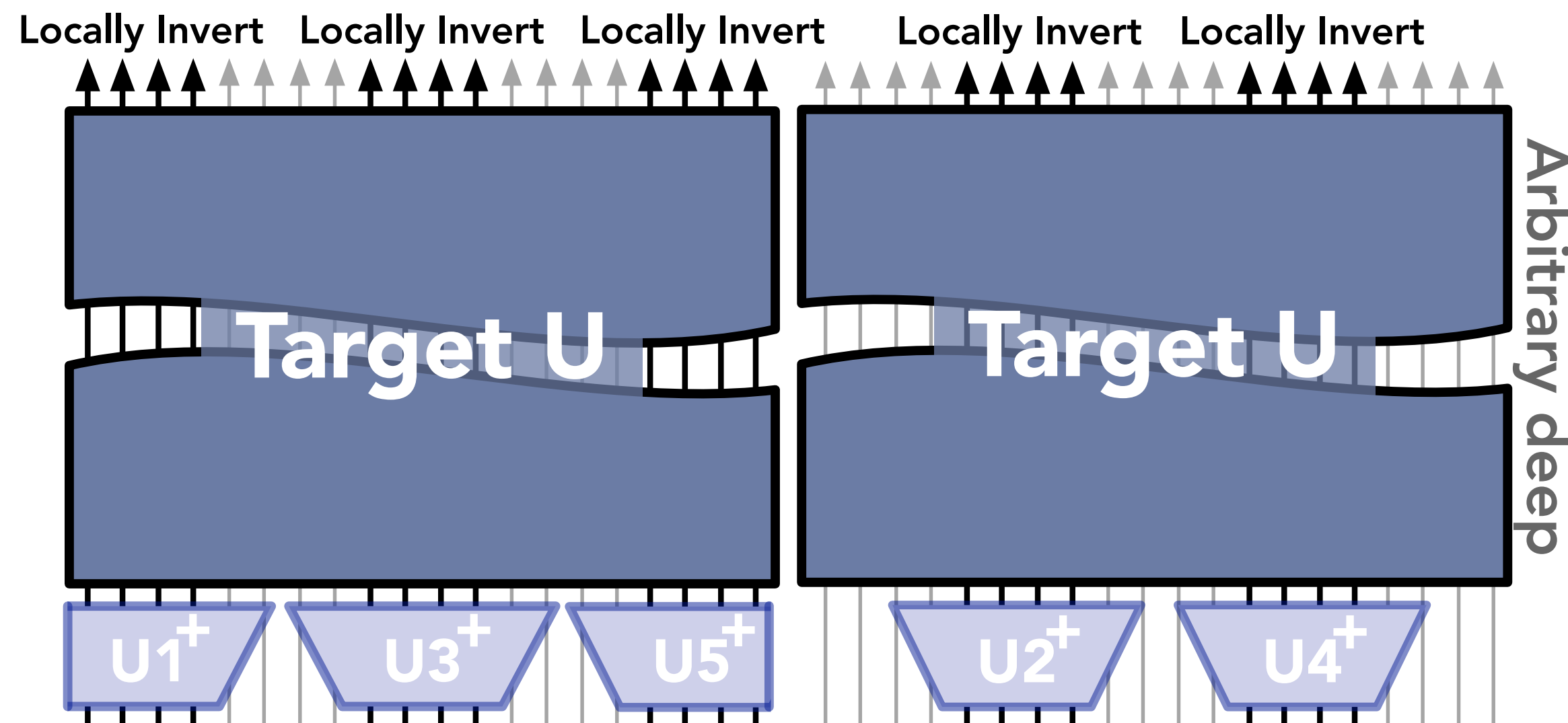


Application: Compressing Dynamics

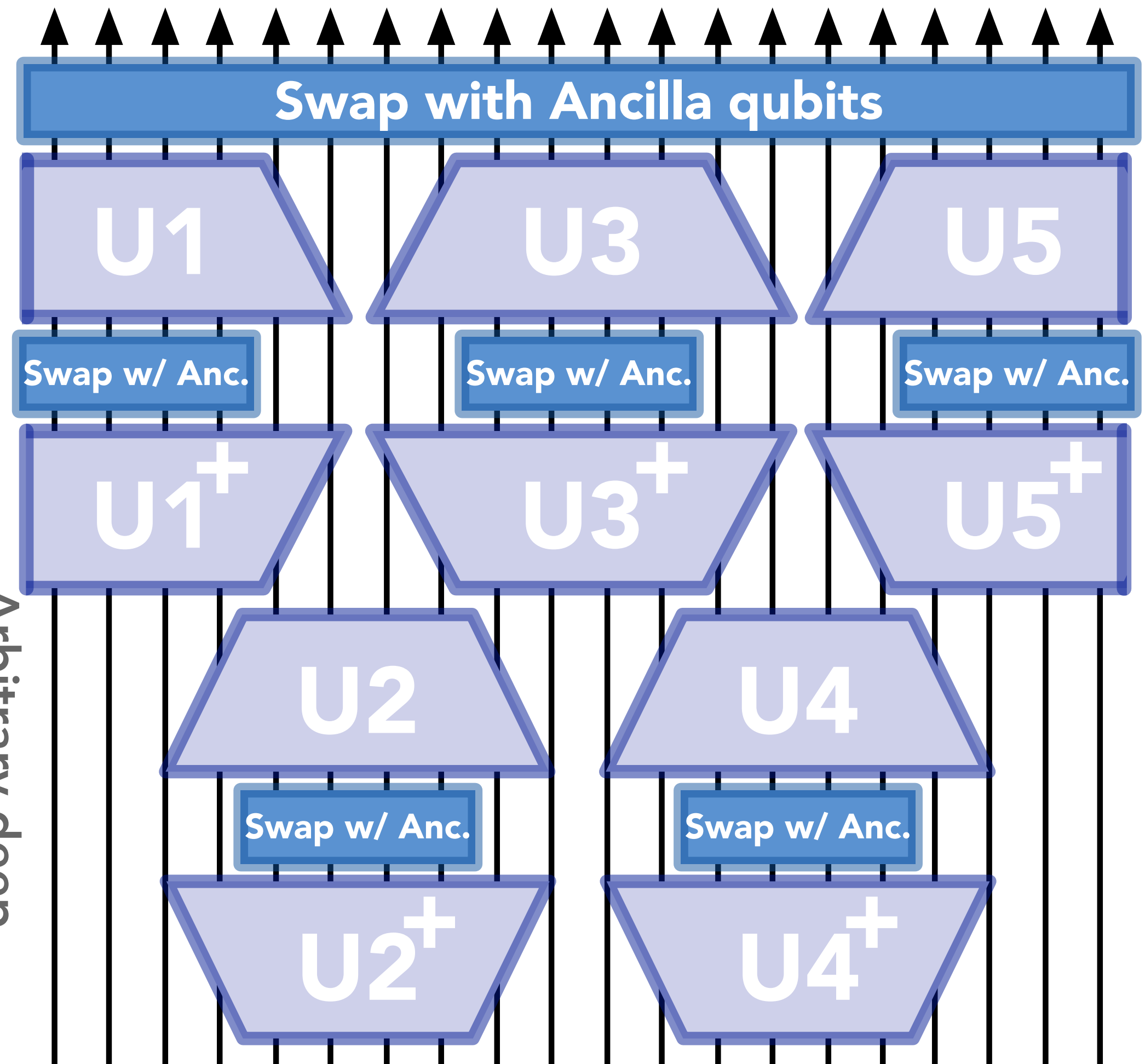
1. Decompose QNN into Pieces



2. Train / Learn Each Pieces



3. Combine the Pieces to form Trained QNN



Application: Compressing Dynamics

